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Topological Analysis of Volatility and Call Option Prices: A Study of Continuity and Monotonicity

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Abstract

This study employs topological analysis to investigate the relationship between volatility and call option prices. Our results reveal a continuous, increasing, and predictable relationship between the two variables, with no distinct clusters or discontinuities. The topological space formed by the call option prices is a simple, connected, and including curve, implying that small changes in volatility result in predictable changes in call option prices. We also, developed and prove three theorems; the Call option price function is continuous with respect to volatility; the Call option price function is monotonically increasing with respect to volatility; and the volatility and Call option price are homeomorphic. These theorems demonstrate the topological properties of the Call option prices and volatility providing new insights into the behavior of option prices. In order to attest to the robustness of our topological analysis, quantile analysis was also considered to understand the proportion of data points below specific values which by implications enables investors and traders, to estimate the probability of call option prices exceeding certain thresholds. These findings have significant implications on option pricing models and risk management strategies, enabling investors and traders to make more informed decisions about option purchases and sales.

Keywords: *Volatility, Stock prices, Continuity, Monotonicity and Topological spaces.*

1. Introduction

The relationship between volatility and call option prices is fundamental aspect of financial market, and understanding its topological features is is crucial for investors, risk managers and financial analysts. Volatility, a measure of the uncertainty or risk associated with the underlying asset, plays a significant role in determining option prices. Call options, which give the holder the right to buy an underlying asset at a specified price, and are sensitive to changes in volatility, and their prices reflect the market's expectations of future price movements. The cost of the option lies on the underlying asset, which is usually a stock, commodity, currency or an index (Kwok, 2008). The holder has the right but cannot be compelled to buy, for European call option. In other words, nonappearance of transactions costs, an in-the money option is always exercised on the expiration date if it has not been exercised earlier, (Hull, 2012). More so, recent researches have shown that implied volatility is a key determinant of option price, and its monotonicity plays a significant role in understanding option price dynamics. Studies have demonstrated that implied volatility tends to increase as the strike price moves away from the at-the-money point, resulting in a volatility smile or skew. The phenomenon has important implications for option pricing and risk management.

However, the relevance of option valuation was first demonstrated by Black-Scholes (Black & Scholes, 1973) when option faced difficulties in valuation of option at expiration. The Black-Scholes equation has been used widely in many financial applications. For instance, (Marcelo et al.,2014) studied implied volatility and implied risk-free rate of return in solving systems of Black-Scholes equations. In the same vein, (Babasola et al., 2008) analyzed Black-Scholes formula for the valuation of European options using Hermite polynomials. Several scholars have written very well on Black-Scholes models such as (Shin & Kim, 2016;

Rodrigo & Mammon,2006; Osu et al.,2009; Osu,2010; Nwobi et al.,2019; Amadi et al.,2024), etc.

The origin of this work lies in the study of (Amadi et al.,2024). This is so, since the path of the stock price method can be allied to his description of the random collision of some tiny particles with the molecules of the liquid he introduced, hence the name Brownian motion. Now, the market price behavior shows the characteristics as a stochastic process called "Brownian motion" or Wiener process with drift. It is an important example of stochastic processes satisfying a Stochastic Differential Equation (SDE) displayed in Figure 1

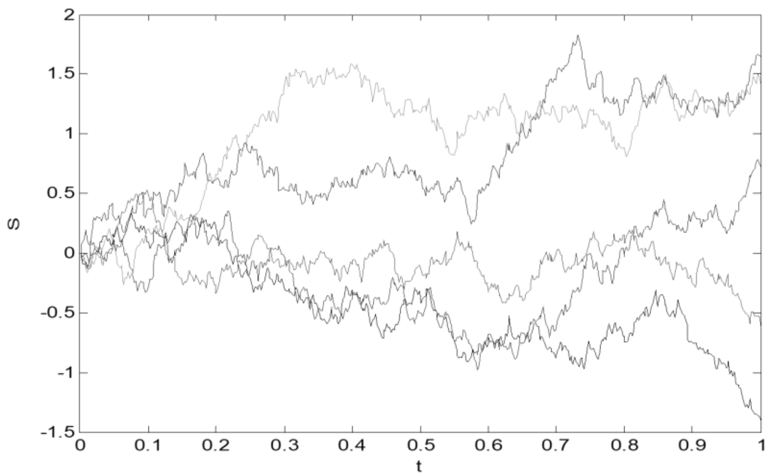

Figure 1: Sample trajectories of the stock price process following Black-Scholes Model

The relationship between volatility and call option prices is a crucial aspect of financial markets, and understanding its topological features can provide valuable insight for investors and risk managers. This study focuses on the topological analysis of volatility and call option prices, exploring the continuity and monotonicity of their relationships. We also, developed and prove three theorems; the Call option price function is continuous with respect to volatility; the Call option price function is monotonically increasing with respect to volatility; and the volatility and Call option price are homeomorphic. These theorems demonstrate the topological properties of the Call option prices and volatility providing new insights into the behaviour of option prices. To validate our analysis, quantile was introduced to provide a comprehensive understanding of the distribution of call option prices and their relationship with volatility. By this analysis, we can gain insight into the probability of a call option price falling within a specific range. To this end, this type of work has not been elsewhere as these widens the area of application of problem of this nature.

For the purpose of this study, the paper is arranged as follows: Section 2.1 presents the Mathematical preliminaries, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

2. Mathematical Preliminaries

Here we present some definitions as foundations of this mathematical finance models.

Definition 1. Probability space: This is a triple $(\Omega, \mathbb{F}, \wp)$ where Ω represents a set of sample space, $\mathbb{F}$ represents a collection of subsets of Ω, while $\wp$ is the probability measure defined on each event $A \in \mathbb{F}$. The collection $\mathbb{F}$ is a σ -algebra or σ -field such as $\Omega \in \mathbb{F}$ and $\mathbb{F}$ is closed under the arbitrary unions and finite intersections. Hence it is called probability measure when the following condition holds.

(i) $P(A) \geq 0 \text{ for all } A \subset \Omega$ (1)

(ii) $P(\Omega) = 1$ (2)

(iii) $A, B \subset \Omega,\ A \cap B = \phi \text{ then } P(A \cup B) = P(A) + P(B)$ (3)

Definition 2. Normal Distribution: A normal distribution function is a peculiar distribution in probability theory and is usually used for modeling asset returns. A normal distribution is used in the Black-Scholes Partial differential equation to value European options. A normal distribution depends on two parameters, (Robert, 1964).

(i) Mean, $\mu \in \mathbb{R}$, is the expectation of a random variable normal distribution.
(ii) Variance, $\sigma^2 > 0$, deals with the magnitude of the spread from the mean.

In Black-Scholes formula, normal distributions are used. The cumulative distribution, usually denoted as $\phi(\mathrm{X})$, is the probability that X will be equal to or less than X ,expressed as $F_x(x) = P(X \leq x)$. A standard normal cumulative distribution function is defined as.

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt \qquad (4)$$

A normal distribution is a symmetric distribution, which means that it touches around a vertical axis of symmetry. Obviously, there is a connection between any given points with same distance to the vertical axis. This relationship is defined in equation (5)

$$\phi(x) = 1 - \phi(-x) \qquad (5)$$

Definition 3. A σ -algebra is a set $\mathbb{F}$ of subsets of Ω with the following axioms:

(i) $\phi, \Omega \in \mathbb{F}$ (6)

(ii) If $A \in \mathbb{F}, then\ A^c \in \mathbb{F}$ (7)

(iii) If $A_1, A_2, \ldots, \in \mathbb{F}, then \bigcup_{k=1}^{\infty} A_k, \bigcap_{k=1}^{\infty} A_k \in \mathbb{F}$ (8)

Clearly $A^c := \Omega - A$ is the complement of A.

Definition 4. If $\mathbb{F}$ is a σ -algebra in Ω, then Ω is called a measurable space and the members of $\mathbb{F}$ are called the measurable sets in Ω.

Definition 5. Let $(\Omega, \mathfrak{M})$ be a measurable space .A map $\mu : \mathfrak{M} \to \mathbb{R} = [0, \infty) \cup \{\infty\}$ is called a measure provided that

(i) $\mu(\phi) = 0$ (9)

(ii) $\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n)$ (10)

***Definition 6. Stochastic process**:* A stochastic process $X(t)$ is a relation of random variables $\{X_t(\gamma), t \in T, \gamma \in \Omega\}$, i.e., for each t in the index set $T, X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t . In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

It can also be seen as a statistical event that evolves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete.

Definition 7. A stochastic process whose finite dimensional probability distributions are all Gaussian.(Normal distribution).

Definition 8. Random Walk: There are different methods to which we can state a stochastic process. Then relating the process in terms of movement of a particle which moves in discrete steps with probabilities from a point $x = a$ to a point $x = b$. A random walk is a stochastic sequence $\{S_n\}$ with $S_0 = 0$, defined by

$$S_n = \sum_{k-1}^{n} X_k \qquad (11)$$

where X_k are independent and identically distributed random variables, (Westergren & Rade 2003).

Definition 9: Stochastic Differential Equation (SDE)

Let $S(t)$ be the price of some risky asset at time t ,and μ ,an expected rate of returns on the stock and dt as a relative change during the trading days such that the stock follows a random walk which is govern by a stochastic differential equation.

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW_t \qquad (12)$$

Where, α is drift and σ the volatility of the stock, W_t is a Brownian motion or Wiener's process on a probability space $(\Omega, \xi, \wp), \xi$ is a $\sigma - \text{algebra}$ generated by $W_t, t \geq 0$.

Theorem 1.1: (Ito's formula) Let $\left(\Omega,\beta,\alpha,\mathrm{F}(\beta)\right)$ be a filtered probability space $X=\{X,t\geq 0\}$ be an adaptive stochastic process on $\left(\Omega,\beta,\alpha,\mathrm{F}(\beta)\right)$ possessing a quadratic variation $\left(X\right)$ with SDE defined as:

$$dX(t)=g\big(t,X(t)\big)dt+f\big(t,X(t)\big)dW(t) \qquad (13)$$

$t\in\Re$ and for $u=u\big(t,X(t)\in C^{1\times 2}(\Pi\times\mathbb{R})\big)$

$$du\big(t,X(t)\big)=\left\{\frac{\partial u}{\partial t}+g\frac{\partial u}{\partial x}+\frac{1}{2}f^{2}\frac{\partial^{2}u}{\partial x^{2}}\right\}d\tau+f\frac{\partial u}{\partial x}dW(t) \quad (14)$$

Nevertheless, the dynamics of option pricing is given by the partial differential equation as follows:

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^{2}S^{2}\frac{\partial^{2}V}{\partial S^{2}}+rS\frac{\partial V}{\partial S}-rV=0 \qquad (15)$$

To eliminate the price process in (15) slightly gives the Black-Scholes analytic formula for Call and Put options:
The analytic formula for the prices of European call option is given as follows

$$\left.\begin{aligned}C&=SN(d_1)-Ke^{-rt}N(d_2)\\ d_1&=\frac{\ln\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\\ d_2&=d_1-\sigma\sqrt{T}\end{aligned}\right\} \qquad (16)$$

where C is Price of a call option, S is price of underlying asset, K is the strike price, r is the riskless rate , T is time to maturity, σ^2 is variance of underlying asset, σ is standard deviation of the (generally referred to as volatility) underlying asset, and N is the cumulative normal distribution.
Similarly, the formula for prices of European put option is given as

$$P = SN\left(d_1\right) - Ke^{-rt}N\left(d_2\right) \qquad (17)$$

Where P is the price of a put option and the meaning of other parameters remain the same as in (16) see (Westergren & Rade,2003; Hull,2003; Higham,2004), and (Hull,2012), etc.

2.1. Mathematical Method of Analysis

Let's denote the volatility as σ and the call option price as $C\left(\sigma\right)$. The method of analysis can be represented mathematically as:
Mapping: Define a mapping $f:\sigma \to C\left(\sigma\right)$ that assigns to each volatility level σ a corresponding call option price $C\left(\sigma\right)$.

Topological space: Consider the topological space $\left(\sum,\tau\right)$ where $\sum$ is the set of volatility levels and τ is the topology induced by the mapping f .
Connectedness: Analyze the connectedness of the space $\left(\sum,\tau\right)$ by examining the continuity of the mapping $f:\sigma \to C\left(\sigma\right)$.

Theorem 2.1 (Continuity of Call option prices): The Call option price function $C:\mathbb{R}+ \to \mathbb{R}+$ is continuous with respect to the volatility $\sigma \in \mathbb{R}+$.

Proof:

Let $\sigma_0 \in \mathbb{R}+$ be a given volatility. We need to show that for any $\varepsilon > 0$ such that $|C\left(\sigma_0\right)| < \varepsilon$ whenever $|\sigma - \sigma_0| < \delta$. Using Black-Scholes model, we have:

$$C\left(\sigma\right) = S_0N\left(d_1\right) - Ke^{(-rt)}N\left(d_2\right)$$

where $d_1 = \left(In\left(S_0 / K\right) + \left(r + \sigma^2 / 2\right)T\right) / \left(\sigma\sqrt{T}\right)$, $d_2 = d_1 - \sigma\sqrt{T}.$
Since $N\left(X\right)$ is a continuous function and d_1 and d_2 are continuous function of σ . We have that $C\left(\sigma\right)$ is a continuous function of σ .
Moreover, using the fact that the Black-Scholes model is a continuous function of σ, We show that $C\left(\sigma\right)$ is continuous function of σ.

Therefore, the option price function C is continuous with respect to the volatility σ.
Monotonicity: Investigate the monotonicity of the mapping $f:\sigma \to C(\sigma)$ by analyzing the behavior of the derivative $dC(\sigma)/d\sigma$.

Theorem 2.2 (Monotonicity of call option): The Call option price function $C:\mathbb{R}+ \to \mathbb{R}+$
is monotonically increasing with respect to the volatility $\sigma \in \mathbb{R}+$

Proof:
Let $\sigma_1, \sigma_2 \in \mathbb{R}+$ with $\sigma_1 < \sigma_2$ .we need to show that $C(\sigma_1) \in C(\sigma_2)$.
Using the Black-Scholes model, we have:

$$\partial C / \partial \sigma = S_0 \sqrt{T} N(d_1) > 0$$

where $N'(X)$ is the derivative of the cumulative distribution function $N(X)$. Since $\partial C / \partial \sigma > 0$. we have that $C(\sigma)$ is a monotonically increasing function of σ . Therefore, the Call option price function C is monotonically increasing with respect to the volatility σ
Topological invariants: Calculate topological invariants, such as persistence diagrams or Betti numbers, to quantify the topological features of the space (Σ, τ).
Topology: Use topological tools, such as persistent homology, to analyze the topological properties of the space (Σ, τ).

Theorem 2.3 (Homeomorphism of volatility and Call option prices): The volatility $\sigma \in \mathbb{R}+$ Call option price $C \in \mathbb{R}+$ are homeomorphic, i.e., There exists a continuous bijection $f:\mathbb{R}+ \to \mathbb{R}+$ such that $f(\sigma) = C$.

Proof:

Since C is a continuous and monotonically increasing function of σ . (by theorem 1 and 2), we can define a function $f:\mathbb{R}+ \to \mathbb{R}+$ by $f(\sigma) = C$. Using the fact that C is a continuous and monotonically increasing function, we can show that f is a homeomorphism. Therefore, the volatility σ and the Call option price C are

homeomorphic. These theorems demonstrate the topological properties of the Call option prices and volatility, which have implications for investment strategies and risk management.
Function Analysis: Apply function analysis techniques such as operator theory, to study the properties of the mapping $f:\sigma \to C(\sigma)$.
The relationship between volatility and call option prices can be represented mathematically using equation such as:

$$C(\sigma) = f(\sigma) \text{ (option pricing function)}$$
$$dC(\sigma)/d\sigma > 0 \text{ (monotonicity condition)} \quad (18)$$

By analyzing the topological properties of the space (Σ, τ) and the mapping $f:\sigma \to C(\sigma)$, we can gain insights into the complex relationship between volatility and call option prices. The above ideas can be seen in the following books (Gunnar and Mikael, 2021), (Herbert and John, 2010), (James and Munkres, 1984), (Erik, 2013).

2.2. Quantile Analysis of Call Option Prices

In order to validate our mathematical analysis, we apply quantile analysis to estimate the probability of a call option price exceeding a certain threshold. Hence, the formula is given as:

$$\text{Quantile} = \frac{(i-0.5)}{n} \quad (1.20)$$

where i is the rank of the observation (from 1 to n), n is the total number of observations, (Azor, 2021).

3. **Results and Discussions**

In this Section we present the computational results for the problem formulated in Section 2.1. The table results are implemented in Matlab programming language.

Table 1: Call option prices and their corresponding varying volatilities with the following parameter values $S_0 = 40,\ K = 25,\ t = 1,$ and $r = 0.2$

Volatility (σ)	**Call Option Prices**
0.25	19.54

0.3	19.57
0.35	19.64
0.4	19.75
0.45	19.91
0.5	20.12
0.55	20.36
0.6	20.64
0.65	20.94
0.7	21.27
0.75	21.62
0.8	21.99

The call option price increases steadily as volatility from 0.25-0.8, the rate of increase in call option price accelerates as volatility approaches higher levels(above 0.6), The relationship between volatility and call option prices appears to be monotonic, meaning that as volatility increases; the call option prices always increases. This situations, means that the positive relationship between volatility and call option prices is consistent with financial theory, which suggests that higher volatility increases the likelihood of extreme price movements, making options more valuable; the accelerating rate of increase in call option price at higher volatility levels suggests that options become increasing sensitive to changes in volatility as volatility increases. Therefore, understanding the impact of volatility on call option prices can help investors and traders make more informed decisions about option purchases and sales, see Table 1.

By applying Subsection 2.1.1 the gives the following topological interpretations:

The data represents a topological space where volatility is the underlying parameter and call option prices are the corresponding values. As volatility increase from 0.25-0.8, the call option prices form a continuous and connected space.

Key features:

- Connectedness: The call option prices form a connected space, meaning that small changes in volatility results in small changes in call option prices.
- Monotonicity: The relationship between volatility and call option prices is monotonic , meaning that as volatility increases, call option prices always increase.

- Smoothness: The data suggests a smooth relationship between volatility and call option prices, with no abrupt changes or discontinuities.

In all, the topological space formed by call option prices is a one dimensional manifold, where each point corresponds to a specific volatility level. The continuous and connected nature of the space implies that small changes in volatility result in predictable changes in call option prices. The topological properties of the call option prices can be used to inform option pricing models and risk management strategies.

Topological Analysis: The data represents a mapping between volatility and call option prices, forming a topological space. Let's analyze the topological features:

- Increasing function: The call option prices increase as volatility increases, indicating a monotonic relationship.
- Continuity: The data points form a continuous curve, suggesting that small changes in volatility result in small changes in call option prices.
- No holes or gaps: The data does not exhibit any holes or gaps, indicating that the topological space is connected.

In general, the topological space is a simple, connected, and continuous curve, where each point corresponds to a specific volatility level and call option price. The monotonicity and continuity of the relationship between volatility and call option prices suggest that topological space is well-behaved and predictable.

- Clustering and sensitivity: The data does not exhibit any clustering or grouping of call option prices, suggesting that this relationship between volatility and call option prices is smooth and continuous.
- Sensitivity: The call option prices are sensitive to changes in volatility, with higher volatility levels resulting in higher call option prices.

In this scenario, the topological analysis provides insight into the behavior of call option prices under different volatility levels. Therefore, understanding the topological features of the relationship between volatility and call option prices can inform option pricing models and risk management strategies.

Table 2: The quantiles of call option prices representing the proportion of data points below specific value

Volatility (σ)	Call Option Price	Quantile
0.25	19.54	0.083
0.3	19.57	0.17
0.35	19.64	0.25
0.4	19.75	0.33
0.45	19.91	0.42
0.5	20.12	0.5
0.55	20.36	0.58
0.6	20.64	0.60
0.65	20.94	0.75
0.7	21.27	0.83
0.75	21.62	0.92
0.8	21.99	1.00

It is clear that in Table 2 ,8.33% of call option prices are below 19.53(corresponding to volatility 0.25), 16.67% of the call option prices are below 19.57 (corresponding to volatility 0.3),50% of the call option prices are below 20.12(corresponding to volatility 0.5), which is the median. Finally,83.33% of call option prices are below 21.27 which corresponds to volatility 0.7

The quantile provides insight into the distribution of call option prices and can be used to estimate the probability of a call option price falling within a specific range. Understanding the quantile of call option prices can help investors and traders to estimate the probability of a call option price exceeding a certain threshold. It will also identify potential trading opportunities based on the distribution of call option prices. To this end, develop risk management strategies that account for the tail behaviour of call option prices.

4. Conclusion

In conclusion, our topological analysis of the relationship between volatility and call option prices reveals a continuous, increasing, and predictable relationship between the two variables. The absence of distinct clusters or discontinuities suggests a smooth and consistent relationship, allowing for accurate predictions of call option prices based on changes in volatility. Our theorems demonstrate that the Call option price function is a continuous and monotonically increasing function of volatility, and that the volatility and Call option price are homeomorphic. However, quantiles analyses were considered which offered a nuanced understanding of the probability of prices falling within specific ranges. The key contribution to knowledge is the establishment of topological framework for analyzing the relationship between volatility and Call option prices, with implications for investment strategies and financial decision making. This study opens up new avenues for research in financial mathematics and computational finance, and has the potential to inform the development of more accurate and robust option pricing models.

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